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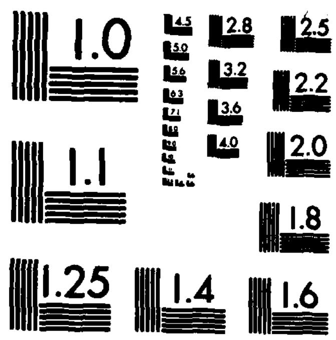

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ACCURATE COMPUTATIONS FOR STEEP SOLITARY WAVES

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ABSTRACT

Finite amplitude solitary waves in water of arbitrary uniform depth are considered. A numerical scheme based on collocation is presented to calculate the highest solitary wave. It is found that the ratio of the amplitude of the wave versus the depth is 0.83322. This value is about 0.006 higher than the values obtained by most previous investigators. In addition another numerical scheme based on an integro-differential formulation is derived to compute solitary waves of arbitrary amplitude. These calculations show that the results of Longuet-Higgins and Fenton (1974) are not accurate for very steep waves. Graphs and tables of the results are included.

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SIGNIFICANCE AND EXPLANATION

Since the time of Scott Russel (1845) many approximate solutions for solitary waves have been obtained. Most of these calculations are in good agreement for relatively small values of the wave height. However, some discrepancies between these calculations appear as the wave of maximum height is approached. In the present paper we present accurate numerical methods to compute steep solitary waves. We show that the ratio of the amplitude of the highest solitary wave versus the depth is 0.83322. This value is about 0.006 higher than the values obtained by most previous investigators.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

ACCURATE COMPUTATIONS FOR STEEP SOLITARY WAVES

J. K. Hunter* and J.-M. Vanden-Broeck**

1. Introduction

Since the time of Scott Russel (1845) many approximate solutions for solitary waves have been obtained. Solutions in the form of an expansion in powers of the wave amplitude were derived by Rayleigh (1876), Korteweg and de Vries (1895), Keller (1948), Laitone (1960), Fenton (1972), Longuet-Higgins and Fenton (1974), Witting (1975) and others. On the other hand direct numerical calculations were attempted by Yamada (1957), Lenau (1966), Yamada, et al (1968), Byatt-Smith (1971) and Byatt-Smith and Longuet-Higgins (1976). A review of these investigations can be found in Miles (1980).

Most of these calculations are in good agreement for relatively small values of the wave height

$$\alpha = \frac{A}{H} \quad . \tag{1.1}$$

Here A is the elevation of the crest of the wave measured from the undisturbed level of the free surface and H is the undisturbed depth.

However, some discrepancies appear as the wave of maximum height is approached. For example the following numerical values for the maximum amplitude α_{max} have been obtained: 0.827 ± 0.008 (Yamada (1957)), 0.827 (Lenau (1966)), 0.8262 (Yamada, et al. (1968)), 0.827 (Longuet-Higgins and Fenton (1974)), 0.8332 (Witting and Bergin, unpublished work mentioned by Witting (1975)) and 0.8332 (Fox, unpublished dissertation mentioned by Schwartz and Fenton (1982)).

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The most reliable calculations for steep solitary waves are those of Longuet Higgins and Fenton (1974) and Byatt-Smith and Longuet-Higgins (1976). Both calculations predict that the highest solitary wave is not the fastest. However the results predicted by these calculations do not agree for very steep waves (see Figure 3). It is worthwhile mentioning that Witting (1975) has suggested that the method used by Longuet-Higgins and Fenton (1974) is defective because the assumed expansion is incomplete.

In this paper we present a numerical scheme based on collocation to compute the solitary wave of maximum height. The method is akin to that of Lenau (1966). However our results are more accurate since we retain up to 100 terms in the power expansion whereas Lenau retained only 9 terms. It is found that $\alpha_{max} = 0.83322$. This value is about 0.006 higher than the values obtained by Yamada (1957), Lenau (1966), Yamada, et al (1968) and Longuet-Higgins and Fenton (1974). On the other hand it agrees with the values mentioned by Witting (1975) and Schwartz and Fenton (1982). We also show that Yamada's (1957) scheme yields the value 0.833 when a sufficiently large number of mesh points is used.

In addition we present another numerical scheme based on an integro-differential equation to compute solitary waves of arbitrary amplitude. The method is similar in philosophy if not in details to the scheme derived by Vanden-Broeck and Schwartz (1979).

Following Longuet-Higgins and Fenton (1974) we introduce the parameter

$$\omega = 1 - \frac{q_c^2}{gH} \quad . \tag{1.2}$$

Here q_c is the velocity at the crest of the wave and g is the acceleration of gravity. The parameter ω varies between 0 and 1 as the wave amplitude varies from zero to its maximum value.

The numerical solutions of our integro-differential equation differ from the results of Longuet-Higgins and Fenton (1974) for $\omega > 0.92$. On the other hand they agree with the numerical results of Byatt-Smith and Longuet-Higgins (1976) for $\omega < 0.96$. Byatt-Smith and Longuet-Higgins (1976) also used an integro-differential formulation. However they were not able to compute waves for $\omega > 0.96$ because too many mesh points were required to describe accurately the flow in the neighborhood of the crest. In the present work this difficulty is avoided by concentrating the mesh points near the crest by an appropriate change of variable. This enables us to compute accurate solutions up to $\omega = 0.99$. An extrapolation of these results shows that $\alpha \to 0.833$ as $\omega \to 1$. This constitutes an important check on the consistency of our two numerical schemes.

The problem is formulated in Section 2 and the highest wave is calculated in Section 3. In Section 4 we compute solitary waves of arbitrary amplitude via an integro-differential equation formulation. The results are discussed in Section 5.

2. Formulation

We consider a two dimensional solitary wave in an inviscid incompressible and irrotational fluid bounded below by an horizontal bottom. We take a frame of reference with the x-axis parallel to the bottom and moving with the phase velocity c of the wave. The level $y = 0$ is chosen as the undisturbed level of the free surface and gravity is assumed to act in the negative y direction.

We introduce the potential function $\phi(x,y)$ and the stream function $\psi(x,y)$. Without loss of generality we choose $\phi = 0$ at the crest and $\psi = 0$ on the free surface. We denote by Q the value of ψ on the bottom. Then

the undisturbed depth H is given by

$$H = \frac{Q}{c} \quad . \tag{2.1}$$

We introduce dimensionless variables by taking H as the unit length and c as the unit velocity. We choose the complex potential

$$f = \phi + i\psi \tag{2.2}$$

as the independent variable.

We shall seek the complex velocity

$$\zeta = u - iv \tag{2.3}$$

as an analytic function of f in the strip $-1 < \psi < 0$. At infinity we require the velocity to be c in the x-direction so that the dimensionless velocity is unity in the x-direction. Therefore ζ must tend to one at infinity.

On the free surface, the Bernoulli equation yields

$$\begin{aligned} \frac{F^2}{2}\,[u^2(\phi) + v^2(\phi)] + \int_0^{\phi} \frac{v(s)}{u^2(s)+v^2(s)}\,ds \\ = \frac{F^2}{2} - \alpha \quad \text{on} \quad \psi = 0 \quad . \end{aligned} \tag{2.4}$$

Here α is the elevation of the crest and F is the Froude number defined by

$$F = \frac{c}{\sqrt{gH}} \quad . \tag{2.5}$$

The functions $u(\phi)$ and $v(\phi)$ in (2.4) denote respectively $u(\phi,0_-)$ and $v(\phi,0_-)$.

On the bottom the kinematic boundary condition yields

$$v = 0 \quad \text{on} \quad \psi = -1 \quad . \tag{2.6}$$

This completes the formulation of the problem of determining the analytic function ζ. This function must tend to one at infinity and satisfy (2.4) on $\psi = 1$ and (2.6) on $\psi = -1$.

Finally let us mention that the asymptotic behavior of $u(\phi) - iv(\phi)$ as $\phi \to \pm\infty$ is described by Stokes' result

$$u(\phi) - iv(\phi) \sim Ae^{-\pi\lambda|\phi|} \quad \text{as} \quad \phi \to \pm\infty \quad . \tag{2.7}$$

Here A is a complex constant to be found as part of the solution and λ is the smallest root of

$$\pi\lambda - \frac{\tan \pi\lambda}{F^2} = 0 \quad . \tag{2.8}$$

3. <u>The highest solitary wave</u>

In this section we present a numerical scheme based on collocation to compute the highest solitary wave. This wave is characterized by a stagnation point at the crest where the surface makes a 120° angle with itself. (See Figure 1.)

Following Lenau (1966) we introduce the new variable t by the relation

$$f = \frac{2}{\pi} \log \frac{1+t}{1-t} - i \quad . \tag{3.1}$$

This transformation maps the flow domain onto the domain $\{|t| < 1, \operatorname{Im} t > 0\}$ in the complex t plane (see Figure 2). We use the notation $t = re^{i\sigma}$ so that the free surface is described by $r = 1$, $0 < \sigma < \pi$.

Lenau (1966) derived the following expansion for the complex velocity ζ

$$\zeta = \left(\frac{1+t^2}{2}\right)^{1/3} e^{\Omega(t)} \tag{3.2}$$

where

$$\Omega(t) = A(1-t^2)^{2\lambda} + \sum_{n=0}^{\infty} a_{n+1} t^{2n} \quad . \tag{3.3}$$

Here λ is the smallest root of (2.8). The coefficients A and a_i ($i = 1,2,3,\ldots$) in (3.3) have to be found to satisfy the boundary condition (2.4).

We solve the problem approximately by truncating the infinite sum in (3.3) after N terms. Differentiating (2.4) with respect to σ and using (3.1) yields

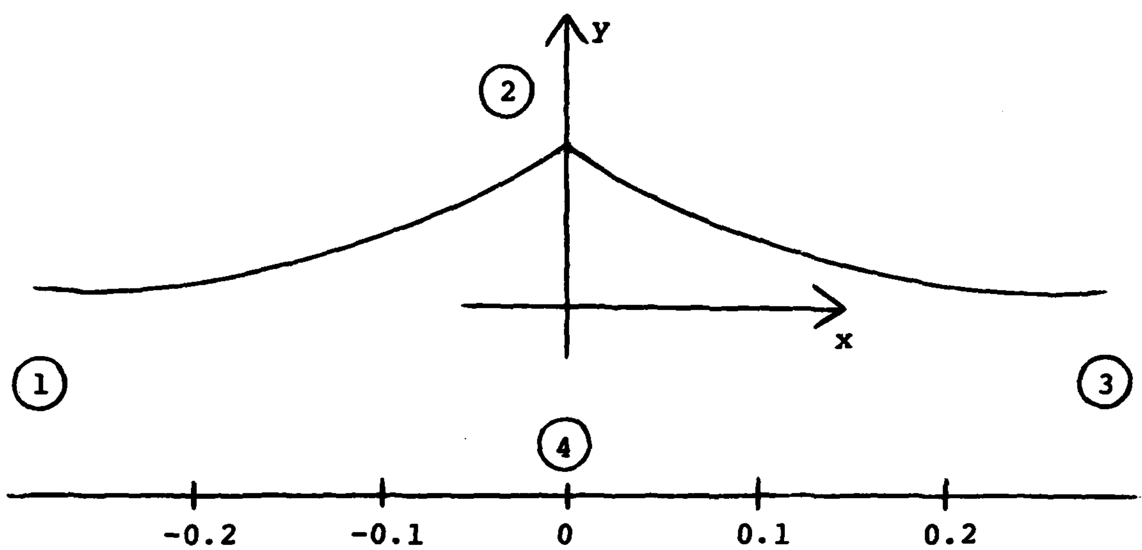

FIGURE 1

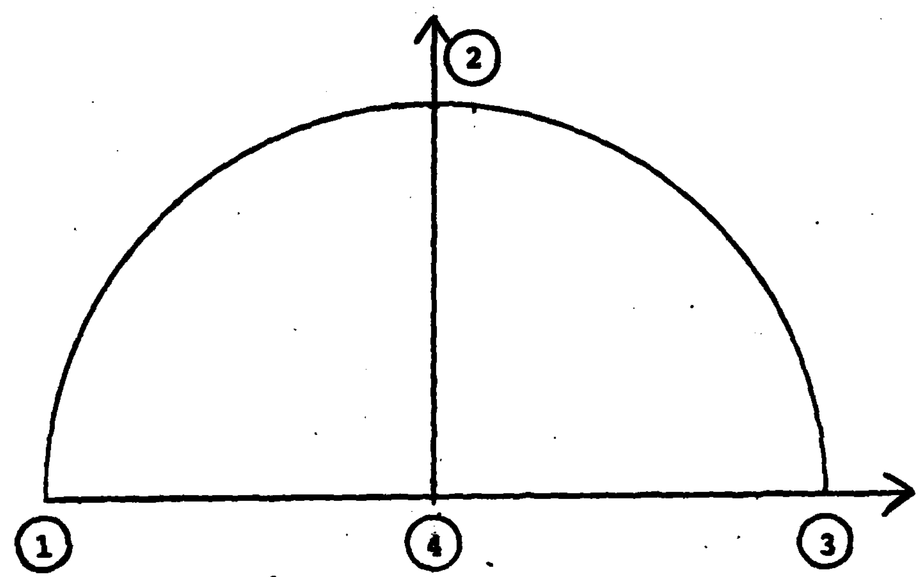

FIGURE 2

$$F^2[\tilde{u}(\sigma)\tilde{u}_\sigma(\sigma) + \tilde{v}(\sigma)\tilde{v}_\sigma(\sigma)]$$

$$- \frac{2}{\pi} \frac{\tilde{v}(\sigma)}{\tilde{u}^2(\sigma)+\tilde{v}^2(\sigma)} \frac{1}{\sin \sigma} = 0 \quad . \tag{3.4}$$

Here $\tilde{u}(\sigma) = u[\phi(\sigma)]$ and $\tilde{v}(\sigma) = v[\phi(\sigma)]$ are the components of the velocity on the free surface.

The functions $\tilde{u}$ and $\tilde{v}$ and their derivatives $\tilde{u}_\sigma$ and $\tilde{v}_\sigma$ are obtained in terms of A, λ and $a_i(i = 1,\ldots,N)$ by substituting $t = e^{i\sigma}$ in (3.2). We find the $N + 3$ unknowns $\lambda, A, F, a_i(i=1,\ldots,N)$ by satisfying (3.4) at the $N + 2$ mesh points

$$\sigma_I = \frac{\pi}{2(N+2)} (I - \frac{1}{2}) \quad I = 1,\ldots,N+2 \quad . \tag{3.5}$$

Thus we obtain a system of $N + 2$ nonlinear algebraic equations. The last equation is obtained by imposing (2.8).

This system of $N + 3$ equations for the $N + 3$ unknowns $\lambda, A, F,$ $a_i(i=1,\ldots,N)$ was solved by Newton's iterations. For most calculations the values $a_1 = -\frac{1}{6}$, $a_i = 0$ $(i=2,\ldots,N)$, $A = -0.32$, $\lambda = 0.32$ and $F = 1.3$ were used as the initial guess. The method converges rapidly and a residual error of 10^{-10} was obtained after 4 or 5 iterations.

Numerical values of F for various values of N are shown in Table 1. These results indicate that the value $F = 1.29091$ is correct to 5 decimal places. The profile of the wave is shown in Figure 1.

Relation (2.4) shows that the amplitude α_{max} of the highest wave is given by

$$\alpha_{max} = \frac{F^2}{2} = 0.83322 \quad . \tag{3.6}$$

This value is about 0.006 higher than the values obtained by Yamada (1957), Lenau (1966), Yamada, et al (1968) and Longuet-Higgins and Fenton (1974). On the other hand it agrees with the values mentioned by Witting (1975) and Schwartz and Fenton (1982).

Table 1: Values of the Froude number of the highest wave for various values of N.

N	F
9	1.28998
15	1.29055
30	1.29083
50	1.29089
75	1.29091
100	1.29091

Our numerical method differs from that of Lenau (1966) because we satisfy (3.5) at the mesh points (3.6) instead of solving for the Fourier coefficients. It is also more accurate because we retain up to 100 terms in (3.3) whereas Lenau retained only 9 terms.

As a further check on our calculations we repeated Yamada's (1957) calculations. Yamada (1957) presented the value 0.827 ± 0.008 obtained with 11 mesh points. With 11 mesh points we also obtained 0.827. However, we obtain 0.832 with 30 mesh points and 0.833 with 100 mesh points. Thus Yamada's (1957) scheme yields the correct answer when a sufficiently large number of mesh points is used.

4. Numerical solution via an integro-differential equation

It is convenient to reformulate the problem as an integro-differential equation by considering $u - iv - 1$. This function tends to zero at infinity. In order to satisfy the boundary condition (2.6) on $\psi = -1$ we reflect the flow in the boundary $\psi = -1$. Thus we seek $u - iv - 1$ as an analytic function of f in the strip $-2 < \psi < 0$.

The values of u and v on the free surface $\psi = 0$ and its image $\psi = -2$ are related by the identities

$$u(\phi,0) = u(\phi,-2) \tag{4.1}$$

$$v(\phi,0) = -v(\phi,-2) \quad . \tag{4.2}$$

In order to find a relation between $u(\phi,0)$ and $v(\phi,0)$ we apply Cauchy's theorem to the function $u - iv - 1$ in the strip $-2 < \psi < 0$. Using (4.1) and (4.2) and exploiting the bilateral symmetry of the wave about $\phi = 0$ we obtain after some algebra

$$u(\phi) - 1 = \frac{1}{p} \int_0^\infty v(s)\left[\frac{1}{s-\phi} + \frac{1}{s+\phi}\right] ds$$

$$+ \frac{1}{\pi} \int_0^\infty \frac{(s-\phi)v(s) + 2[u(s)-1]}{(s-\phi)^2 + 4}\, ds \tag{4.3}$$

$$+ \frac{1}{\pi} \int_0^\infty \frac{(s+\phi)v(s) + 2[u(s)-1]}{(s+\phi)^2 + 4}\, ds \quad .$$

The first integral in (4.3) is of Cauchy principal value form. We shall measure the amplitude of the wave by the parameter ω. Using the symmetry of the wave about $\phi = 0$, we rewrite (1.2) in the form

$$\omega = 1 - F^2[u(0)]^2 \quad . \tag{4.4}$$

Using (4.4) and (2.4) evaluated at $\phi = 0$ we obtain

$$\alpha = \frac{F^2}{2} + \frac{\omega-1}{2} \quad . \tag{4.5}$$

For a given value of ω, (2.4), (4.3) and (4.5) define a system of integro-differential equatins for $u(\phi)$, $v(\phi)$, α and F.

In order to solve these equations we find it convenient to introduce the new variable β instead of ϕ by the relation

$$\phi = \beta^\gamma, \ \gamma > 1 \quad . \tag{4.6}$$

Therefore we rewrite (2.4), (4.3) and (4.5) in terms of β, $u^*(\beta) = u[\phi(\beta)]$ and $v^*(\beta) = v[\phi(\beta)]$.

Next we introduce the M mesh points

$$\beta_I = (I-1)E \qquad I = 1,\ldots,M \tag{4.7}$$

where E is the interval of discretization. The change of variable (4.6) is chosen because it concentrates the mesh points near the crest of the wave. For very steep waves the value of γ was taken as 3.

We shall satisfy (2.4) and (4.3) at the points $\beta_{I+\frac{1}{2}} = \frac{1}{2}(\beta_I + \beta_{I+1})$ $I = 1,\ldots,M-1$. Thus we obtain, after discretization $2M - 2$ nonlinear algebraic equations for the $2M + 2$ unknowns α, F, and $u^*(\beta_I)$, $v^*(\beta_I)$ $I = 1,\ldots,M$. Relations (4.4) and (4.5) provide two more equations. An extra

equation is obtained by imposing the symmetry condition

$$v^*(\beta_1) = 0 \quad . \tag{4.8}$$

The last equation expresses $u^*(\beta_M)$ in term of $u^*(\beta_{M-1})$ and $u^*(\beta_{M-2})$ by an extrapolation formula based on the asymptotic formula (2.7). The discretization of (2.4) and (4.3) follows closely the work of Vanden-Broeck and Schwartz (1979).

The system of $2M + 2$ equations was solved by Newton's iterations.

The most important source of error in the numerical scheme arises from the truncation of the infinite integrals in (4.3) at

$$s = \phi_{max} = [(M-1)E]^{\gamma} \quad . \tag{4.8}$$

We used two different methods to approximate the infinite integrals in (4.3). In the first method we used the asymptotic formula (2.7) to approximate the integrals between ϕ_{max} and infinity. This approach is similar to the method used by Byatt-Smith and Longuet-Higgins (1976). In the second method we simply neglected the contribution of the integrals between ϕ_{max} and infinity. In this second method we also replaced the equation in which an extrapolation based on (2.7) is used by a Lagrange extrapolation formula. Thus the second method is completely independent of (2.7). Both methods were found to give accurate results. However the first method is more efficient because accurate results can be obtained with ϕ_{max} relatively small. Most of the results presented in the next section were obtained by using the first method.

5. Discussion of the results

In the first calculation the iterations were started with the classical solution of the Korteweg and de Vries equation. For ω small the iterations converged rapidly. Once a solution was obtained it was used as an initial guess for a larger value of ω and so on.

For each value of ω we took E small enough and ϕ_{max} large enough for the results to be independent of E and ϕ_{max}. This was achieved in the following way. For a given value of ϕ_{max} we progressively decreased E to a value for which the results were independent of E to the degree of accuracy desired. We repeated the procedure for larger and larger values of ϕ_{max} up to a value for which the results were also independent of ϕ_{max}. This procedure is illustrated in Table 2.

In Figure 3 we present the numerical values of the Froude number F versus ω. These results confirm that the highest solitary wave is not the fastest. We also show the results obtained by Longuet-Higgins and Fenton (1974) and by Byatt-Smith and Longuet-Higgins (1976). Our results agree with those of Longuet-Higgins and Fenton for $\omega < 0.92$ and with those of Byatt-Smith and Longuet-Higgins for $\omega < 0.96$.

Byatt-Smith and Longuet-Higgins (1976) were not able to compute waves for $\omega > 0.96$ because their numerical procedure uses equal increments in the velocity potential. This is not well suited to the calculation of very steep waves because large curvature, low velocity and sparse point spacing are characteristic of the crest region. In the present work this difficulty has been avoided by concentrating the mesh points near the crest by the change of variable (4.6).

Table 2: Values of F when $\omega = 0.98$ and $\gamma = 3$

(a) $\phi_{max} = 7$		(b) $\phi_{max} = 10$	
N	F	N	F
100	1.29141	100	1.29395
120	1.29143	120	1.29152
150	1.29145	150	1.29145
180	1.29145	180	1.29145

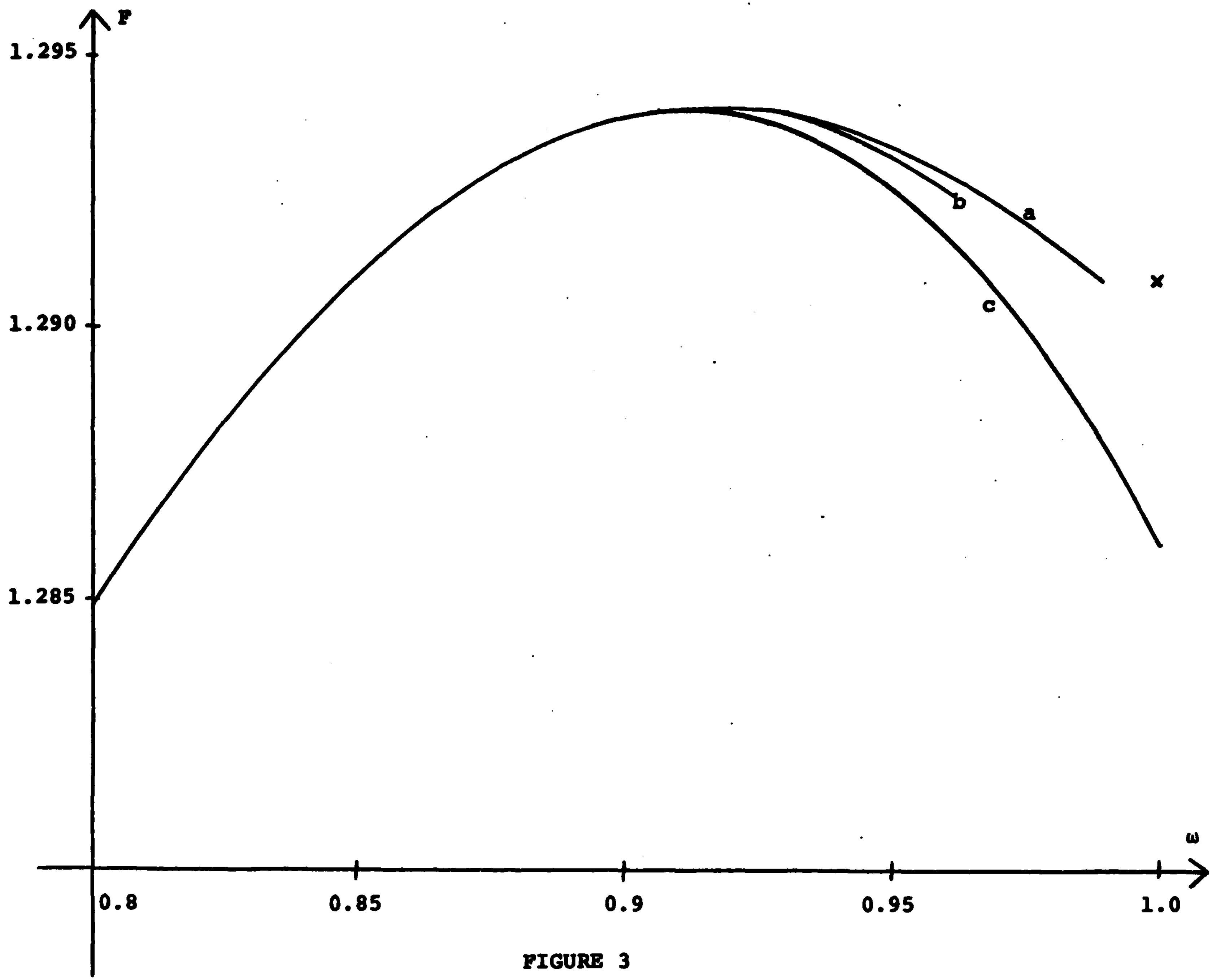

FIGURE 3

Our calculations show that the results of Longuet-Higgins and Fenton (1974) are not accurate for $\omega > 0.92$. This seems to confirm Witting's (1975) suggestion that their method is defective because the assumed expansion is incomplete.

Acknowledgement

The authors are indebted to Professor P. G. Saffman for suggesting the calculation described in the last paragraph of section 3.

Captions for Figures

Figure 1. Computed free-surface profile for the highest solitary wave. The vertical scale is the same as the horizontal scale.

Figure 2. Flow configuration in the complex t-plane

Figure 3. The Froude number F as a function of ω as given by the numerical scheme of Section 4 (curve a), Byatt-Smith and Longuet-Higgins (1976) (curve b) and Longuet-Higgins and Fenton (1974) (curve c). The cross corresponds to the highest wave calculated in Section 2.

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